Chapter 3: Adjustment Costs in the Labour Market

I. Motivational Questions and Exercises:

Exercise 3.1 (p. 106):

Illustrate the derivation of equation (3.5) of the textbook.

Solution: The intertemporal marginal product of labour is represented by

$$\text{(3.1)} \qquad \lambda_t = E_t\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i\left(\mu\left(Z_{t+i}, N_{t+1}\right)-w\right)\right],$$

where $\mu\left(Z_t, N_t\right)$ is the immediate marginal product of labour, r is the real interest rate, Z denotes the product demand, N is the level of employment, $E_t[\cdot]$ is the expectation operator based on the information set available at t, and w is the wage. Using dynamic programming, we can rewrite (3.1) as follows:

$$\text{(3.2)} \qquad \lambda_t = \mu\left(Z_t, N_t\right)-w+E_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i\left(\mu\left(Z_{t+i}, N_{t+i}\right)-w\right)\right],$$

If we re-index $\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i\left(\mu\left(Z_{t+i}, N_{t+i}\right)-w\right)$ from $i = 0$, instead of $i = 1$, we have $\left(\frac{1}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i\left(\mu\left(Z_{t+1+i}, N_{t+1+i}\right)-w\right)$, and therefore

$$\text{(3.3)} \qquad \lambda_t = \mu\left(Z_t, N_t\right)-w+\left(\frac{1}{1+r}\right)E_t\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i\left(\mu\left(Z_{t+1+i}, N_{t+1+i}\right)-w\right)\right].$$

By the law of iterative expectations, the above equation can be written as

$$\text{(3.4)} \qquad \lambda_t = \mu\left(Z_t, N_t\right)-w+\left(\frac{1}{1+r}\right)E_t\left[E_{t+1}\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i\left(\mu\left(Z_{t+1+i}, N_{t+1+i}\right)-w\right)\right]\right].$$

By (3.1), we can define λ_{t+1} as follows

$$\text{(3.5)} \qquad \lambda_{t+1} = E_{t+1}\left[\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i\left(\mu\left(Z_{t+1+i}, N_{t+1+1}\right)-w\right)\right]$$

Substituting (3.5) into (3.4) gives

(3.6) $$\lambda_t = \mu(Z_t, N_t) - w + \left(\frac{1}{1+r}\right)E_t[\lambda_{t+1}],$$

which is equation (3.5) in the textbook.

Exercise 3.2 (p. 125):

Consider the two-state Markov process

(3.6) $$x_{t+1} = \begin{cases} a & \text{with prob } p \text{ if } x_t = a; \text{ with prob } (1-p) \text{ if } x_t = b \\ b & \text{with prob } q \text{ if } x_t = b; \text{ with prob } (1-q) \text{ if } x_t = a \end{cases}.$$

What is the expected duration of the Markov process in state *a*?

<u>Solution:</u> Given that the process is currently in state *a*, *p* is the transition probability for the Markov process to start and stay in state *a*, and *D* is the duration of state *a* we have

(3.7) $$\begin{aligned} &D = 1, \text{ if } x_t = a \text{ and } x_{t+1} \neq a; \text{ prob}(D=1) = (1-p) \\ &D = 2, \text{ if } x_t = x_{t+1} = a \text{ and } x_{t+2} \neq a; \text{ prob}(D=2) = p(1-p) \\ &D = 3, \text{ if } x_t = x_{t+1} = x_{t+2} = a \text{ and } x_{t+3} \neq a; \text{ prob}(D=3) = p^2(1-p) \\ &\vdots \end{aligned}$$

The expected duration of state *a* can then be derived as

(3.8) $$\begin{aligned} &E(D) = \sum_{a=1}^{\infty} a \,\text{prob}(D=a) \\ &+ 1 \times \text{prob}(x_{t+1} \neq a \mid x_t = a) \\ &+ 2 \times \text{prob}(x_{t+1} = a,\, x_{t+2} \neq a \mid x_t = a) \\ &+ 3 \times \text{prob}(x_{t+1} = a,\, x_{t+2} = a,\, x_{t+3} \neq a \mid x_t = a) \\ &+ \cdots \\ &\vdots \\ &= 1 \times (1-p) + 2 \times p(1-p) + 3 \times p^2(1-p) + \cdots \\ &= \frac{1}{(1-p)} \end{aligned}$$

Exercise 3.3 [Blanchard and Fisher (1989), Problem 3, p. 491]:

Consider a firm and a union who bargain with each other over wages and employment. Wages are chosen so as to maximise $N(w - A)[R(N) - wN]$ subject to the 'right-to-manage' constraint, $R'(N) = w$. N is employment, $R(N)$ is the revenue function, and A is the expected level of income for those who are not employed by the firm.

(a) Derive the wage level resulting from the Nash bargain as a function of the labour income to profits ratio, θ, the wage elasticity of labour demand, η, and A.

(b) Let A be given by $\overline{w}(1-u) + Bu$ where u is the rate of unemployment, $\overline{w}$ the outside wage if employed elsewhere, and B the income if unemployed. Derive the rate of unemployment consistent with a symmetric equilibrium, assuming for convenience that $B/\overline{w}$ is constant and denoted by ρ.

Solution:

(a) Consider the "right-to-manage" model in which the wage is determined in a non-cooperative Nash bargain and in which the firm subsequently chooses optimal labour demand from the labour demand curve. The objective function of the union is *N*(*w*-*A*), while the objective function of the firm is [*R*(*N*) - *wN*]. Consequently, the Nash bargaining solution is determined by

(3.9) $$\max_w N(w-A)[R(N)-wN]-\lambda[R'(N)-w]$$

We get the following two first-order conditions:

(3.10) $$[N_w(w-A)+N][R(N)-wN]+N(w-A)[N_w R'(N)-N-wN_w]-\lambda[R'_w-1]=0$$

and

(3.11) $$R'(N)=w,$$

Equation (…) implies $R'_w(N)=1$. We therefore get

(3.12) $$[N_w(w-A)+N][R(N)-wN]+N(w-A)[N_w R'(N)-N-wN_w]=0$$

Since $R'(N)=w$, we have $N_w R'(N)-N-wN_w = N_w[R'(N)-w]-N=-N$. Thus,

(3.13)
$$\begin{aligned}
&[N_w(w-A)+N][R(N)-wN]=N^2(w-A)\\
\Leftrightarrow &\left[\frac{N_w}{N}(w-A)+1\right][R(N)-wN]=N(w-A)\\
\Leftrightarrow &\left[-\frac{\eta}{w}(w-A)+1\right]\frac{wN}{\theta}=N(w-A)\\
\Leftrightarrow &\left[-\frac{\eta}{w}(w-A)+1\right]\frac{1}{\theta}=\frac{w-A}{w}\\
\Leftrightarrow &\frac{1}{\theta}=\frac{w-A}{w}\left(1+\frac{\eta}{\theta}\right)\\
\Leftrightarrow &\frac{w-A}{w}=\frac{1}{\eta+\theta}
\end{aligned}$$

where we have used the definitions of the wage elasticity of labour demand $\eta=-N_w w/N$ and the labour income to profit share $\theta=wN/(R(N)-wN)$. When η is increasing, the unions anticipate the larger impact of higher wages upon employment and therefore the wage level will be lower. In the limit for $\eta\to\infty$, we get $w = A$.

(b) Inserting $A=\overline{w}(1-u)+Bu=\overline{w}[(1-u)+\rho]=\overline{w}[1-(1-\rho)u]$, where $\rho=B/\overline{w}$ into the real wage equation and recognising $R'(N)=w$, we arrive at

$$\frac{w-A}{w}=\frac{w-\left[(1-u)\overline{w}+uB\right]}{w}$$

(3.14) $$\Leftrightarrow \frac{w-A}{w}=\frac{(1-u)w+uw-\overline{w}\left[(1-u)+u\rho\right]}{w}$$

$$\Leftrightarrow \frac{w-A}{w}=(1-u)\frac{w-\overline{w}}{w}+u\left(1-\frac{B}{w}\right)$$

A higher $\rho = B/\overline{w}$ and therefore a more generous unemployment benefit system will increase equilibrium unemployment. In a symmetric equilibrium we have $w=\overline{w}$ and therefore equation (3.14) simplifies to

(3.15) $$\frac{w-A}{w}=u(1-\rho)\ .$$

The corresponding equilibrium unemployment rate is

(3.16) $$u=\left(\frac{w-A}{w}\right)\left(\frac{1}{1-\rho}\right).$$

Additional Reference:

Blanchard, O.J. and S. Fisher (1989) *Lectures on Macroeconomics*, Cambridge (MIT Press).

II. Software Tools

Software Exercise 3.1: Explore the cyclical behaviour of *Z*, *L*, and intertemporal marginal value of profits in exercise 23 on p. 113 and pp. 244-245

Solution: The starting points of hiring and firing decisions are illustrated by the equations (*) and (**) on p. 244:

(3.17) $$h=\int_t^{\infty}\left[Z_\tau L_\tau^{-\beta}-w\right]e^{-r(\tau-t)}d\tau\ ,$$

(3.18) $$-f=\int_t^{\infty}\left[Z_\tau L_\tau^{-\beta}-w\right]e^{-r(\tau-t)}d\tau\ ,$$

where L is the number of employees, r is discount rate, $\beta>0$ represents the concavity of production function, h is the marginal cost of hiring, f is the marginal cost of firing, and Z denotes the deterministically fluctuating level of demand $Z(\tau)=K_1+K_2\sin\left(\frac{2\pi}{p}\tau\right)$, where $K_1>K_2>0$. The values of K_1, K_2, and p determine the magnitude and the length of the business cycle. It is assumed that employees do not quit and that there are no quadratic costs of hiring and firing – the firm hires or fires immediately so that $\int_t^{\infty}\left[Z_\tau L_\tau^{-\beta}-w\right]e^{-r(\tau-t)}d\tau$ will never be greater than h or lower than $-f$. The inaction area is defined by the regime of $-f<\int_t^{\infty}\left[Z_\tau L_\tau^{-\beta}-w\right]e^{-r(\tau-t)}d\tau<h$.

For the inaction area, where no hiring/firing happens, the integral $v = \int_t^{\infty}\left[Z_\tau L_\tau^{-\beta} - w\right]e^{-r(\tau-t)}d\tau$ can be written as follows:

$$(3.19)\quad v = \frac{K_1 L^{-\beta}}{r} - \frac{w}{r} + K_2 L^{-\beta}\int_t^{\infty}\sin\left(\frac{2\pi}{p}\tau\right)e^{-r(\tau-t)}d\tau\,,$$

where $v = \int_t^{\infty}\left[Z_\tau L_\tau^{-\beta} - w\right]e^{-r(\tau-t)}d\tau$. The integral formula on p. 245 of the textbook can be used to solve $\int_t^{\infty}\sin\left(\frac{2\pi}{p}\tau\right)e^{-r(\tau-t)}d\tau$:

$$(3.20)\quad \int \sin(\gamma x)e^{\lambda x}d\tau = \frac{\lambda e^{\lambda x}}{\gamma^2+\lambda^2}\left(\sin(\gamma x) - \frac{\gamma}{\lambda}\cos(\gamma x)\right).$$

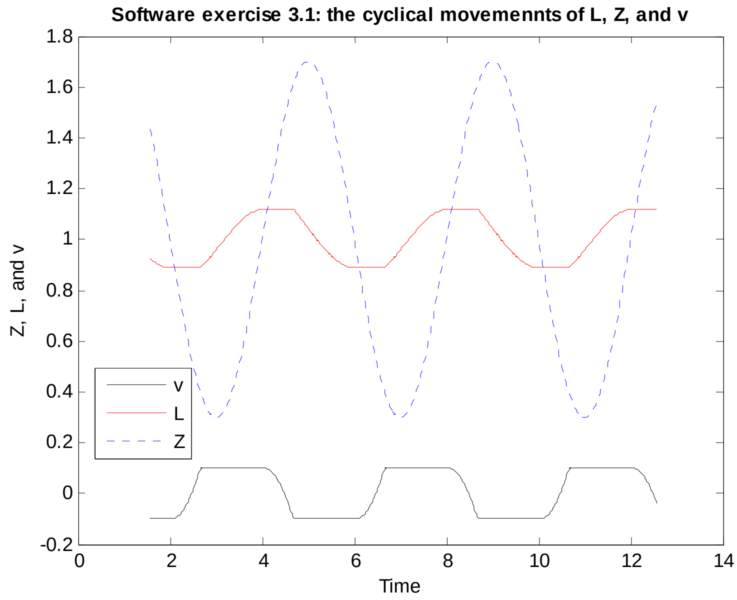
Substituting the above equation into (3.19) yields

$$(3.21)\quad v = \frac{K_1 L^{-\beta}}{r} - \frac{w}{r} + K_2 L^{-\beta}\left[\frac{re^{\lambda x}}{\gamma^2+r^2}\left(\sin\left(\frac{2\pi}{p}x\right) + \frac{2\pi}{pr}\cos\left(\frac{2\pi}{p}x\right)\right)\right].$$

For $v(L) > h$, the firm immediately hires $L_h - L$ so that $v(L_h) = h$. The newly hired workers, $L_h - L$, lead to a lower marginal product of labour so that $v = h$ again. Similarly, for $v(L) < -f$, the firm immediately lays off $L - L_f$ so that $v(L_f) = -f$. The firing of $L - L_f$ employees leads to a higher marginal product of labour so that $v = -f$ again.

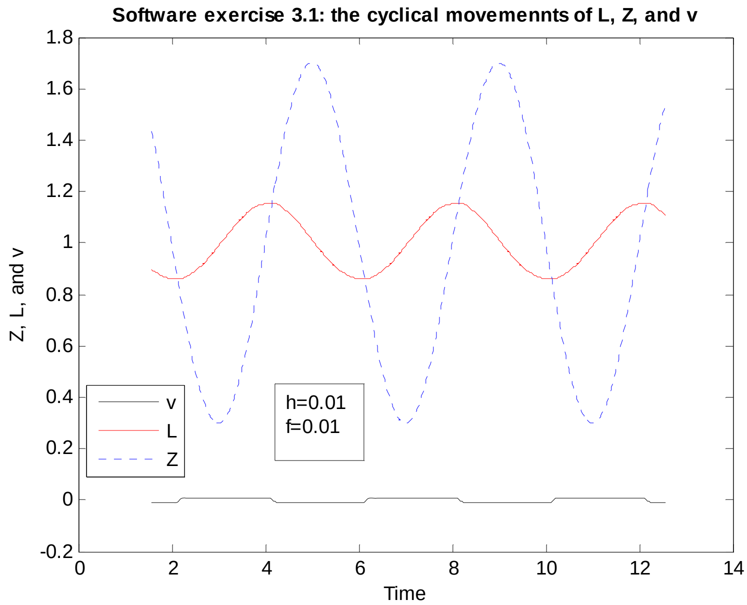
The Matlab file, “main_Exercise_3-1.m”, allows to analyse such a system of *L* fluctuations due to hiring and firing over the deterministic business cycles, which is denoted by the sin function in Z.

```
% beginning of input data for (2.276) and (2.277)
r     = 0.1;
w     = 1.00;
L     = 1.00; %initial value of L
beta  = 0.3;
h     = 0.10;
f     = 0.10;
K1    = 1.00;
K2    = 0.50;
p     = 4.00;
% end of input data
```

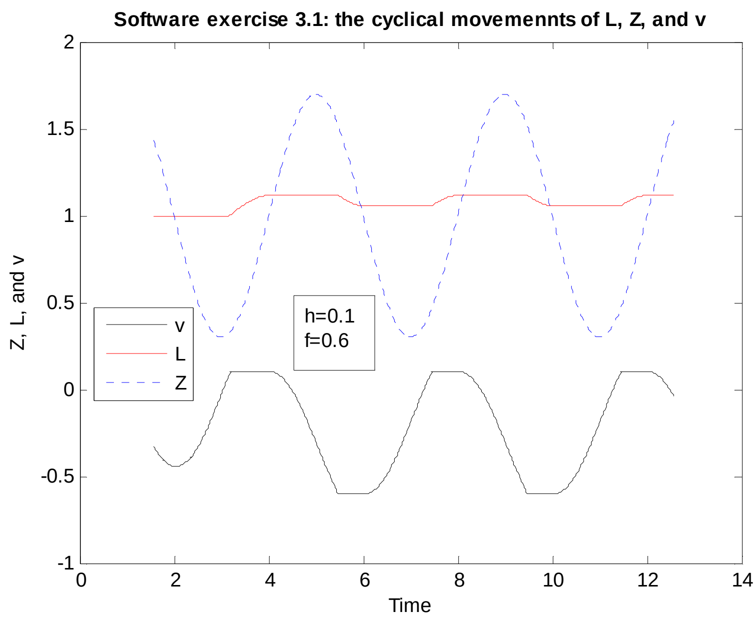


Note that there are some differences between the time series of Z and v since v is the intertemporal value. The peaks and troughs of Z do not coincide with the maximum points of v. The reason is that v will only fluctuate in the areas of $-f \leq v \leq h$. When v is beyond the peak, the firm stops hiring. Conversely, the firm stops firing after v reaches its trough.

If hiring and firing costs are very small, say h = 0.01 and f = 0.01, then the fluctuations of Z will mostly lead to variations in L, not in v.



On the contrary, if h = 0.1 and there exist substantial firing costs, say f = 0.6, then the fluctuations of Z will mostly lead to bigger variations in v, but smaller fluctuations in L.



Software-Exercise 3.2: Demonstrate the impacts of changes in the quit rate, firing costs, and uncertainty upon the hiring and firing thresholds in exercise 24 on p. 114 and pp. 246-247

Solutions: The solutions for *v* in the differential equation on p. 246 are denoted by

(3.22) $$v = \frac{\eta}{r-\theta+(1-\beta)\delta} - \frac{w}{r+\delta} + K_1\eta^{\alpha_1} + K_2\eta^{\alpha_2},$$

where $\eta = ZL^{-\beta}$, *L* is the number of employees, *r* is the interest rate, δ is the quit rate, θ is the drift parameter of geometrical Brownian motions of demand *Z*, σ is the standard deviation, *w* is the wage level, and K_1 and K_2 are parameters related to diffusion processes and had to be solved by the value-matching and smooth-pasting conditions. Note that there is a typo in the equation for the particular solution in the textbook!

Let Z_+ and Z_- represent the hiring and firing thresholds. The value-matching conditions derived using the dynamic programming approach are denoted by

(3.23) $$\frac{Z_+L^{-\beta}}{r-\theta+(1-\beta)\delta} - \frac{w}{r+\delta} + K_1\left(Z_+L^{-\beta}\right)^{\alpha_1} + K_2\left(Z_+L^{-\beta}\right)^{\alpha_2} = H,$$

and

(3.24) $$\frac{Z_-L^{-\beta}}{r-\theta+(1-\beta)\delta} - \frac{w}{r+\delta} + K_1\left(Z_-L^{-\beta}\right)^{\alpha_1} + K_2\left(Z_-L^{-\beta}\right)^{\alpha_2} = -F,$$

where *H* is the marginal hiring cost and *F* is the marginal firing cost. The corresponding smooth-pasting conditions are

(3.25) $$\frac{L^{-\beta}}{r-\theta+(1-\beta)\delta} + \alpha_1K_1Z_+^{\alpha_1-1}L^{-\beta\alpha_1} + \alpha_2K_2Z_+^{\alpha_2-1}L^{-\beta\alpha_2} = 0,$$

and

$$(3.26)\quad \frac{L^{-\beta}}{r-\theta+(1-\beta)\delta}+\alpha_1 K_1 Z_-^{\alpha_1-1}L^{-\beta\alpha_1}+\alpha_2 K_2 Z_-^{\alpha_2-1}L^{-\beta\alpha_2}=0\,.$$

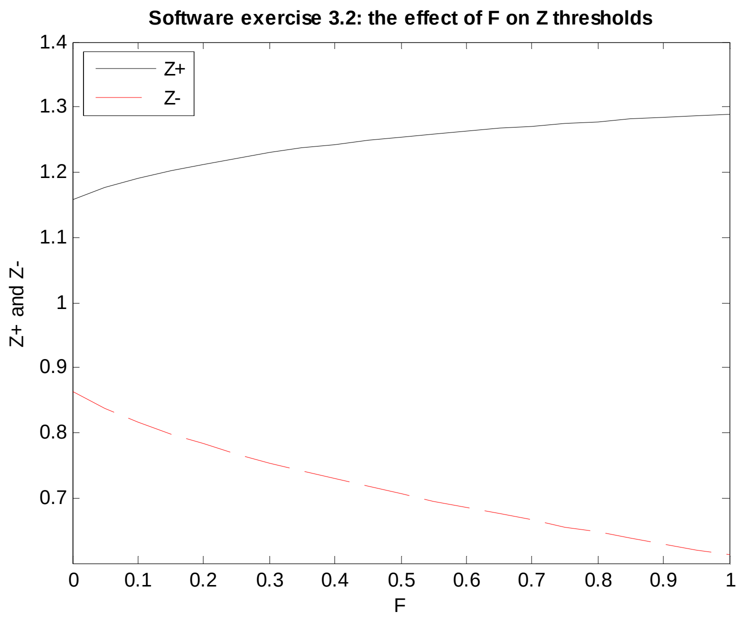
Equations (3.25) and (3.26) can be solved numerically using the Matlab m files "fun_Exercise_3_2.m" and "main_Exercise__2.m". The input data are as follows:

```
% beginning of input data
r     = 0.1;
beta  = 0.30;
delta = 0.05;
theta = 0.02;
sigma = 0.20;
L     = 1.00;
wage  = 1.00;
H     = 0.1;
F     = 0.6;
choiceofplot = 1; % "1" for plot of F;
                  % "2" for plot of sigma;
                  % "3" for plot of delta, for exercise 2.3 (c);
% end of input data
```

Given the choice of the value of choiceofplot, 1 or 2 or 3, the Matlab m files will generate a plot that shows the effect of *F* or σ or δ on the hiring thresholds Z_+ and the firing thresholds Z_-.

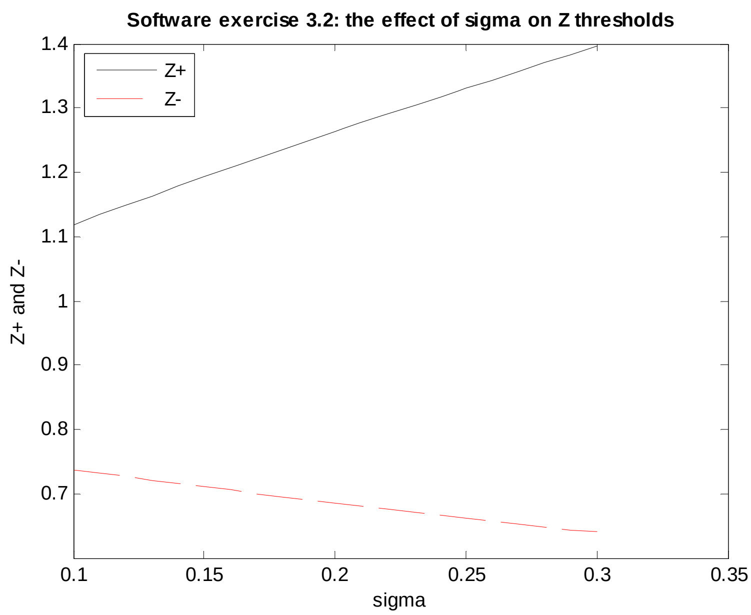
The Effect of F on Thresholds

As *F* increases, the hiring thresholds fall – the firms are more reluctant to fire marginal employees. The effect of *F* on hiring thresholds works indirectly via the diffusion term $K_2\left(Z_+L^{-\beta}\right)^{\alpha_2}$. Higher *F* leads to a lower value of $K_2\left(Z_-L^{-\beta}\right)^{\alpha_2}$ at the firing thresholds and also lowers $K_2\left(Z_+L^{-\beta}\right)^{\alpha_2}$ at the hiring thresholds. Therefore, the hiring thresholds are higher for rising *F*.



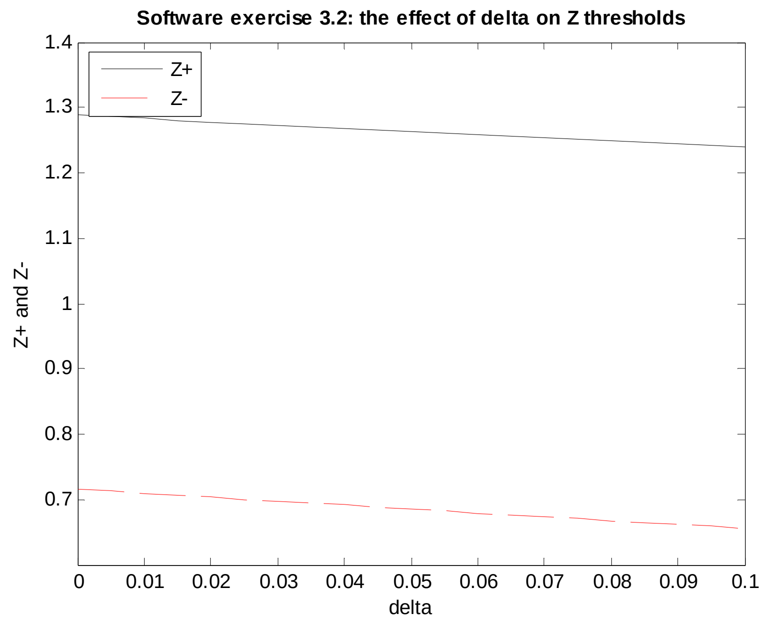
The Effect of σ on Thresholds

The value of σ only affects the diffusion terms of the solutions. As expected, a riskier environment (higher σ) widens the inaction area. Thus, a rise in σ makes the firm reluctant to hire or fire marginal employees.



The Effect of δ on Thresholds

As δ increases, the particular integral falls and leads to higher hiring and firing thresholds. However, a rising δ has a negative impact on the option (diffusion) terms and leads to lower thresholds. As indicated by the graph below, the sum of the diffusion terms $K_1\left(ZL^{-\beta}\right)^{\alpha_1} + K_2\left(ZL^{-\beta}\right)^{\alpha_2}$ is negative.



Software-Exercise 3.3: Working time and employment decisions – An extension of software exercise 3.2

Assume that a firm has the following immediate profit:

$$\Pi = \frac{1}{1-\beta} Z\left(Lg(H)\right)^{1-\beta} - \left[w(H) + x\right]L$$

where x is the fixed costs of employment, g is a function related to working hours, wage, w, is also a function of working hours, H denotes working hours, and all other variables are the same as in software exercise 3.2. Z again follows a geometrical Brownian motion. The existence of fixed costs per worker x tends to make firms demand longer working hours in order to spread these costs over more hours of work. g and w have the following functional forms:

$$g(H) = \begin{cases} H_S^{\gamma}\left(H / H_S\right)^{\delta} & H > H_s \\ H^{\gamma} & H \leq H_s \end{cases}$$

$$w(H) = \begin{cases} w_S H_s + a w_s \left(H - H_S\right) & H > H_s \\ w_S H & H \leq H_s \end{cases}$$

It is assumed that $0 < \delta < 1$ so that *g(H)* is strictly concave and the problem of the firm is well defined. An exogenous reduction of standard hours H_s may increase or decrease *g(H)* and – depending on the overtime wage premium – increase or decrease employment and labour services. The firm pays a constant premium $a > 1$ on overtime hours ($H - H_s$). The marginal costs of hiring and firing are denoted by T and F respectively, and the employees never quit. The firm simultaneously chooses actual hours and employment to maximise its expected discounted value of profits.
(a) Derive the Bellman equation of the above set-up and solve the working time employment decisions set-up.
(b) Analyse numerically the impact of the overtime wage premium a, firing costs, F, and hiring costs, T, on the employment thresholds and hours worked.

Solution:

(a) The firm's expected value of discounted profits without any firing and/or hiring costs is

$$(3.27)\quad V = \max_{L,H} E\left[\int_0^\infty \left[\frac{1}{1-\beta} Z\left(Lg(H)\right)^{1-\beta} - \left[w(H)+x\right]L\right] e^{-rs} ds\right],$$

where r is the real rate of interest and $E[\cdot]$ is the expectation operator. According to equation (3.27), the firm chooses how many people to employ and the specific number of hours, given the wage schedule. Using Itô´s Lemma, the Bellman equation for the value V at time zero in the continuation region is

$$(3.28)\quad rV = \max_{L,H}\left\{ Z\left(Lg(H)\right)^{1-\beta} - \left[w(H)+x\right]L + \eta Z V_Z + \frac{1}{2}\sigma^2 Z^2 V_{ZZ}\right\}.$$

The first term on the right-hand side is revenue, $\left[w(H)+x\right]L$ is the employment-related bill, $\eta Z V_Z$ is the gain due to a Z shock, and the last term is the change in the value of the firm caused by changes in demand. The first-order conditions for H are:

$$(3.29)\quad (1-\beta) Z L^{1-\beta} g^{-\beta}(H) g'(H) - w'(H) L = 0$$

After solving the above equation, the variable H becomes a function of Z given the functions of w and g. The first-order condition with respect to L is denoted by

$$(3.30)\quad rv = Z L^{-\beta} g^{1-\beta}(H) - \left[w(H)+x\right] + \eta Z v_Z + \frac{1}{2}\sigma^2 Z^2 v_{ZZ}$$

where $v = V_L$ is the value of employing the marginal worker. As in the previous exercise, the inaction area for hiring and firing is determined by the condition

$$(3.31)\quad -F < v < T$$

The hiring thresholds, Z_+ , are determined when $v = T$ and the firing thresholds, Z_-, happen when $-F = v$. In the absence of hiring and firing costs, the particular integral may be expressed as

$$(3.32)\quad v^P(Z) = E\left[\int_0^\infty \left[Z L^{-\beta} g^{1-\beta}(H) - \left[w(H)+z\right]\right] e^{-rs} ds\right] = \frac{Z L^{-\beta} g^{1-\beta}(H)}{r-\theta} - \frac{w(H)+z}{r}.$$

The firm's option value of hiring in the future and its option value of firing once the worker is employed are measured by the homogenous part of the equation

$$(3.33)\quad rv = \theta Z v_Z + \frac{1}{2}\sigma^2 Z^2 v_{ZZ}.$$

The hiring options are denoted by $A_1 Z^{\alpha_1}$ and the firing options are represented by $A_2 Z^{\alpha_2}$, where A_1 and A_2 are parameters to be determined by the boundary conditions and α_1 and α_2 are the positive and negative roots of the following characteristic equation:

(3.34) $\frac{1}{2}\sigma^2\alpha(\beta-1)+\theta\alpha-r=0$.

The value-matching conditions of hiring and firing follow:

(3.35) $$\frac{Z_+L^{-\beta}g^{1-\beta}(H)}{r-\theta}-\frac{w(H)+x}{r}+A_2Z_+^{\alpha_2}=T+A_1Z_+^{\alpha_1}$$

and

(3.36) $$-\left[\frac{Z\_L^{-\beta}g^{1-\beta}(H)}{r-\theta}-\frac{w(H)+x}{r}\right]+A_1Z_-^{\alpha_1}=F+A_2Z_-^{\alpha_2}.$$

The left-hand sides of (3.35) and (3.36) show the marginal benefit from hiring/firing a worker and the right-hand sides give the corresponding marginal costs. The smooth-pasting conditions ensure that hiring (firing) is not optimal either before or after the hiring (firing) threshold is reached:

(3.37) $$\frac{L^{-\beta}g^{1-\beta}(H)}{r-\theta}+A_2\alpha_2Z_T^{\alpha_2-1}=+A_1\alpha_1Z_T^{\alpha_1-1}$$

and

(3.38) $$-\frac{L^{-\beta}g^{1-\beta}(H)}{r-\theta}+A_1\alpha_1Z_-^{\alpha_1-1}=+A_2\alpha_2Z_-^{\alpha_2-1}.$$

Equations (3.35) - (3.38) form a non-linear system of equations with four unknown parameters, Z_+, Z_-, A_1, and A_2, and can be solved numerically once the solutions for α_1 and α_2 are obtained from (3.34) and optimal values for *H* are found for the values of Z_T and Z_F via equation (3.29):

(3.39) $$(1-\beta)Z_+L^{1-\beta}g^{-\beta}(H)g'(H)-w'(H)L=0$$

and

(3.40) $$(1-\beta)Z\_L^{1-\beta}g^{-\beta}(H)g'(H)-w'(H)L=0$$

for hiring and hiring thresholds respectively.

Equations (3.35) - (3.40) can be solved numerically using the following Matlab m files:

- “fun_Exercise_3_2.m” for value-matching and smooth-pasting conditions
- “fun_Exercise_3_2_g.m” for function $g(H)$
- “fun_Exercise_3_2_g_d.m” for the derivative of $g(H)$
- “fun_Exercise_3_2_w.m” for function $w(H)$
- “fun_Exercise_3_2_w_d.m” for the derivative of $w(H)$
- “fun_Exercise_3_2_Hh.m” for determination of hours in hiring thresholds
- “fun_Exercise_3_2_Hf.m” for determination of hours in firing thresholds
- “main_Exercise_3_3.m” for the main program

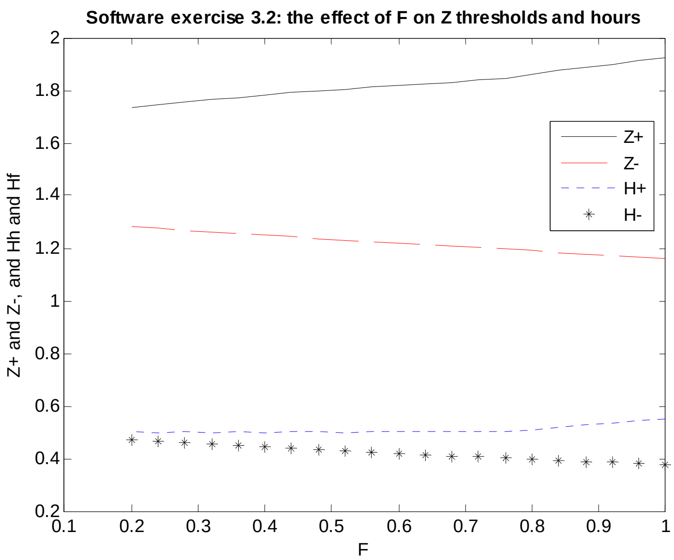
The input data are as follows:

```
% beginning of input data
r     = 0.08;  % interest rates
beta  = 0.30;  % for CD function
theta = 0.0;   % drift parameter for geometrical Brownian motions
sigma = 0.13;  % risk parameter for geometrical Brownian motions
L     = 1.0;   % the level of employees
T     = 0.1;   % marginal hiring cost
F     = 0.6;   % margianl firing cost
a     = 1.4;   % premium rate for over time
HS    = 0.5;   % benchmark value for hours
WS    = 1.0;   % benchmark value for wages
gama  = 0.8;   % for wage function
delta = 0.8;   % for wage function
xx    = 0.5;  % x, the fixed cost related wages.
choiceofplot = 3; % "1" for plot of F;
                  % "2" for plot of T;
                  % "3" for plot of "a" for exercise 2.3 (c);
% end of input data
```

Given the choice of the value of choiceofplot, 1 or 2 or 3, the Matlab m files will generate a plot that shows the effect of *F* or *T* or *a* on the hiring thresholds Z_+ and the firing thresholds Z_-, and corresponding hours.

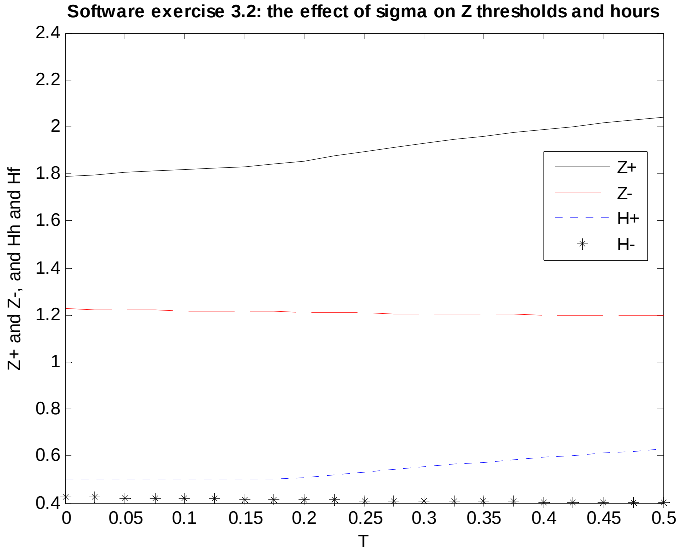
The Effect of F on Thresholds

The effect of *F* on the hiring and firing thresholds are similar to the ones in software exercise 3.2, except the inaction area is wider. As *F* increases, the firm lowers the working hours further before firing; the rise in *F* also leads to higher hiring thresholds since the firm opts instead for increasing working hours.



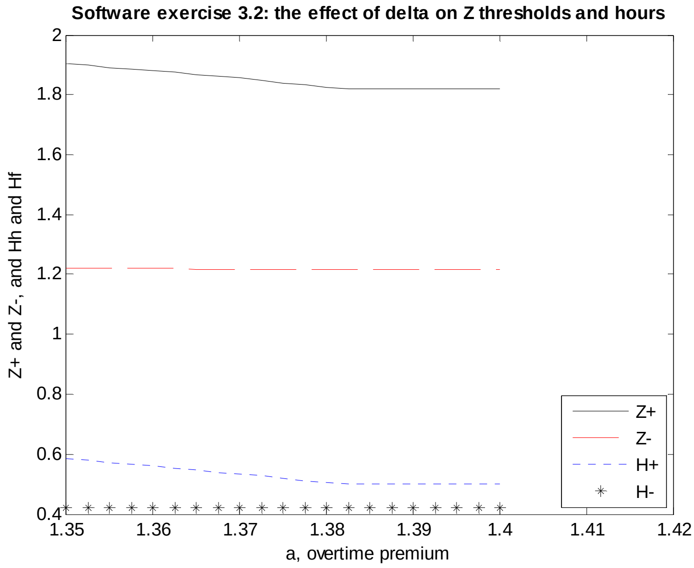
The Effect of T on the Thresholds

The increase in *T* has a direct impact on hiring thresholds, amplified by the higher working hours introduced to offset rising hiring costs. The impact of changes in *T* on firing thresholds is minimal due to smaller firing options (from high firing costs in the benchmark value).



The Effect of the Overtime Premium (a) on the Thresholds

For lower overtime premiums, the firm prefers increasing working hours rather than hiring additional employees. The impact of changes in *a* on the firing thresholds is smaller since the overtime premium only directly affects the wage function in hiring decisions.



<u>Additional Reference:</u>

Chen, Y.-F. and Funke, M. (2004) „Working Time and Employment Under Uncertainty“, *Studies in Nonlinear Dynamics and Econometrics* 8, Issue 3, Article 5, (www.bepress.com/snde/vol8/iss3/art5).